1. The inclusion-exclusion principle is a principle in combinatorics that allows us to calculate the size of a set that is a union of several disjoint sets. It states that the size of the union of two or more sets can be calculated by adding the sizes of the individual sets and then subtracting the sizes of their pairwise intersections, and so on.

Let's use this principle to solve the given problem:

Let A be the set of people who speak Hindi, and let B be the set of people who speak English.

We are given:

|A| = 35 (number of people who speak Hindi)

|A ∩ B| = 25 (number of people who speak both English and Hindi)

|A ∪ B| = 50 (total number of people in the group)

We want to find:

1. The number of people who speak only English and not Hindi (|B - A|).

2. The number of people who speak English (|B|).

Using the inclusion-exclusion principle:

|A ∪ B| = |A| + |B| - |A ∩ B|

Substituting the given values:

50 = 35 + |B| - 25

Simplifying the equation:

|B| = 50 - 35 + 25

|B| = 40

Therefore, there are 40 people who speak English.

To find the number of people who speak only English and not Hindi (|B - A|), we can subtract the number of people who speak both English and Hindi (|A ∩ B|) from the total number of people who speak English (|B|):

|B - A| = |B| - |A ∩ B|

|B - A| = 40 - 25

|B - A| = 15

Therefore, there are 15 people who speak only English and not Hindi.

2.

To simplify the expression and express it in the form x + iy, we can use the properties of complex numbers and Euler's formula:

First, let's simplify the numerator and denominator separately.

Numerator: (cos θ + isin θ)^5

Using De Moivre's theorem, we can expand this as:

(cos θ + isin θ)^5 = cos(5θ) + isin(5θ).

Denominator: (cos θ - isin θ)^4

Similarly, using De Moivre's theorem, we can expand this as:

(cos θ - isin θ)^4 = cos(4θ) - isin(4θ).

Now, let's divide the numerator by the denominator:

z = [(cos 5θ + isin 5θ) / (cos 4θ - isin 4θ)]

To simplify further, we multiply the numerator and denominator by the conjugate of the denominator to eliminate the imaginary part in the denominator:

z = [(cos 5θ + isin 5θ) \* (cos 4θ + isin 4θ)] / [(cos 4θ - isin 4θ) \* (cos 4θ + isin 4θ)]

z = [(cos 5θ + isin 5θ) \* (cos 4θ + isin 4θ)] / [(cos^2 4θ + sin^2 4θ)]

z = [(cos 5θ + isin 5θ) \* (cos 4θ + isin 4θ)] / 1

z = (cos 5θ \* cos 4θ - sin 5θ \* sin 4θ) + i(cos 5θ \* sin 4θ + sin 5θ \* cos 4θ)

z = cos(5θ + 4θ) + i sin(5θ + 4θ)

z = cos(9θ) + i sin(9θ)

Now the expression z = cos(9θ) + i sin(9θ) is in the x + iy form.

To find the modulus (absolute value) of z, we can use the property of complex numbers:

|z| = sqrt(x^2 + y^2)

|z| = sqrt(cos^2(9θ) + sin^2(9θ))

|z| = sqrt(1)

|z| = 1

Therefore, the modulus of z is 1.

To find the amplitude (argument) of z, we can use the inverse tangent function:

amplitude = atan2(y, x)

amplitude = atan2(sin(9θ), cos(9θ))

amplitude = 9θ

Therefore, the amplitude of z is 9θ.

3.

(a).

To evaluate the integral ∫(0 to π/2) √(1 + sin^2x) dx, we'll start by using a trigonometric identity:

1 + sin^2x = cos^2x

∫(0 to π/2) √(cos^2x) dx

Since , cos ^ 2x is always non-negative in the given interval, we can remove the square root:

∫(0 to π/2) cosx dx

Now, let's integrate cosx with respect to x:

∫ cosx dx = sinx + C

Where C is the constant of integration. Now, evaluate the integral limits:

∫(0 to π/2) cosx dx = [sin(π/2) - sin(0)] = [1 - 0] = 1

So, the correct value of the integral ∫(0 to π/2) √(1 + sin^2x) dx is 1.

(b)

To solve the given first-order differential equation:

(2x - y + 1)dx + (2y - x - 1)dy = 0

We can check whether it's exact or not by computing the partial derivatives of the two terms with respect to x and y:

∂/∂y (2x - y + 1) = -1

∂/∂x (2y - x - 1) = -1

Since both partial derivatives are equal, the differential equation is exact. To find the solution, follow these steps:

Step 1: Find the potential function (also called the integrating factor).

The integrating factor (IF) is a function μ(x, y) that makes the differential equation exact. It's found by the following formula:

μ(x, y) = e^(∫ (∂/∂y) of (2x - y + 1) dx)

μ(x, y) = e^(∫ (-1) dx) = e^(-x)

Step 2: Multiply both sides of the equation by the integrating factor μ(x, y) = e^(-x):

e^(-x)(2x - y + 1)dx + e^(-x)(2y - x - 1)dy = 0

Step 3: Rewrite the left side as the exact derivative of a product:

d(e^(-x)(2x - y + 1)) = 0

Step 4: Integrate both sides with respect to their respective variables:

∫d(e^(-x)(2x - y + 1)) = ∫0 dy

Step 5: Integrate the left side with respect to y and the right side with respect to y:

e^(-x)(2x - y + 1) = C1

where C1 is the constant of integration.

Step 6: Solve for y:

y = 2x + 1 - e^(-x) + C1 \* e^(-x)

where C1 is an arbitrary constant.

4.

a. To check whether the propositions ~(p ∧ q) and (∼p) ∨ (∼q) are logically equivalent, we can construct truth tables for both propositions and compare their results.

truth table for ~(p ∧ q):

|  |  |  |  |
| --- | --- | --- | --- |
| p | q | P ^ q | ~(p ^ q) |
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| f | f | F | T |

truth table for (∼p) ∨ (∼q):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | ~p | ~q | ~(p ) ^(~ q) |
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

compare the truth values for both propositions:

~(p ∧ q) | (∼p) ∨ (∼q) | Equivalence?

|  |  |  |
| --- | --- | --- |
| ~(p ^ q) | (~p) v (~q) | Equivalence |
| F | F | Yes |
| T | T | Yes |
| T | T | Yes |
| T | T | Yes |

As we can see from the truth table, for all possible combinations of truth values of p and q, ~(p ∧ q) and (∼p) ∨ (∼q) have the same truth values. Therefore, the propositions are logically equivalent.

b.

To construct the multiplication table for the group G under multiplication modulo 18, we need to perform the multiplication operation for each pair of elements in the set G.

Recall that multiplication modulo 18 means that after performing the multiplication, we take the remainder when divided by 18.

Here's the multiplication table for the group G:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| \* | 1 | 5 | 7 | 11 | 13 | 17 |
| 1 | 1 | 5 | 7 | 11 | 13 | 17 |
| 5 | 5 | 7 | 11 | 13 | 17 | 1 |
| 7 | 7 | 11 | 13 | 17 | 1 | 5 |
| 11 | 11 | 13 | 17 | 1 | 5 | 7 |
| 13 | 13 | 17 | 1 | 5 | 7 | 11 |
| 7 | 7 | 1 | 5 | 7 | 11 | 13 |

Next, let's find the inverse of each element in the group G. The inverse of an element a in a group is an element b such that a \* b = b \* a = 1 (identity element).

From the multiplication table above, we can see the inverse of each element:

- The inverse of 1 is 1 (1 \* 1 = 1 mod 18).

- The inverse of 5 is 11 (5 \* 11 = 11 \* 5 = 1 mod 18).

- The inverse of 7 is 13 (7 \* 13 = 13 \* 7 = 1 mod 18).

- The inverse of 11 is 5 (11 \* 5 = 5 \* 11 = 1 mod 18).

- The inverse of 13 is 7 (13 \* 7 = 7 \* 13 = 1 mod 18).

- The inverse of 17 is 17 (17 \* 17 = 1 mod 18).

Every element in the group G has an inverse, making G a group under multiplication modulo 18.

5

(a)

Length of the Arc (s):

Length of the arc (s) = r \* θ

where r is the radius of the circle and θ is the angle in radians.

Area of the Sector (A):

Area of the sector (A) = (1/2) \* r^2 \* θ

where r is the radius of the circle and θ is the angle in radians.

Given:

Radius (r) = 2 cm

Angle (θ) = 20 radians

Let's calculate the length of the arc (s) first:

Length of the Arc (s) = r \* θ

Length of the Arc (s) = 2 cm \* 20 radians

To find the area of the sector, we'll use the formula:

Area of the Sector (A) = (1/2) \* r^2 \* θ

Area of the Sector (A) = (1/2) \* (2 cm)^2 \* 20 radians

Now, let's calculate both values:

Length of the Arc (s):

Length of the Arc (s) = 2 cm \* 20 radians

Length of the Arc (s) = 40 cm (rounded to 2 decimal places)

Area of the Sector (A):

Area of the Sector (A) = (1/2) \* (2 cm)^2 \* 20 radians

Area of the Sector (A) = (1/2) \* 4 cm^2 \* 20 radians

Area of the Sector (A) = 40 cm^2 \* 20 radians

Area of the Sector (A) = 800 cm^2 (rounded to 2 decimal places)

So, the length of the arc formed by an angle of 20 radians in a circle with a radius of 2 cm is approximately 40 cm, and the area of the sector formed by the same angle is approximately 800 cm².

(b)

lim┬(n⟶∞)⁡〖(2+n+n^2)/(2+3n+4n^2)〗

Divide both numerator and denominator by n^2:

lim┬(n⟶∞)⁡〖(2/n^2 + n/n^2 + 1)/(2/n^2 + 3n/n^2 + 4)〗

As n approaches infinity, all terms with 1/n^2 become negligible, and the expression simplifies to:

lim┬(n⟶∞)⁡〖(0 + 0 + 1)/(0 + 0 + 4)〗

Now, we can calculate the limit:

lim┬(n⟶∞)⁡〖1/4 = 1/4〗

So, the limit of the expression as n approaches infinity is 1/4.

To find the limit of the expression as x approaches 2, we can directly substitute the value of x into the expression. However, if the resulting expression yields an indeterminate form (0/0 or ±∞/∞), we can use algebraic manipulation or L'Hôpital's rule to solve the limit.

Let's begin by substituting x = 2 into the expression:

lim┬(x⟶2)⁡〖(2x^2 - 3x - 2)/(x - 2)〗

Substitute x = 2:

(2 \* 2^2 - 3 \* 2 - 2) / (2 - 2)

Now, simplify the expression:

(2 \* 4 - 3 \* 2 - 2) / 0

(8 - 6 - 2) / 0

(0) / 0

We get an indeterminate form, 0/0. Now, we can apply L'Hôpital's rule. To do this, we take the derivative of the numerator and the derivative of the denominator separately and then calculate the limit of their ratio.

Derivative of the numerator:

d/dx (2x^2 - 3x - 2) = 4x - 3

Derivative of the denominator:

d/dx (x - 2) = 1

Now, we have:

lim┬(x⟶2)⁡〖(2x^2 - 3x - 2)/(x - 2)〗 = lim┬(x⟶2)⁡〖(4x - 3)/1〗

Substitute x = 2:

lim┬(x⟶2)⁡〖(4 \* 2 - 3)/1 = 5/1 = 5〗

So, the limit of the expression as x approaches 2 is 5.

6.

(a)

To find the derivative of the given function y = (x + sin(x))/(e^x - cos(x)), we'll use the quotient rule of differentiation. The quotient rule states that if we have a function in the form of f(x) = u(x)/v(x), then the derivative f'(x) is given by:

f'(x) = (v(x) \* u'(x) - u(x) \* v'(x)) / [v(x)]^2

where u'(x) is the derivative of u(x) with respect to x, and v'(x) is the derivative of v(x) with respect to x.

Let's proceed with finding the derivative of the given function step by step:

Identify u(x) and v(x).

u(x) = x + sin(x)

v(x) = e^x - cos(x)

Find u'(x) and v'(x).

u'(x) = d/dx (x + sin(x)) = 1 + cos(x) (derivative of x is 1, derivative of sin(x) is cos(x))

v'(x) = d/dx (e^x - cos(x)) = e^x + sin(x) (derivative of e^x is e^x, derivative of -cos(x) is sin(x))

Apply the quotient rule.

y' = [(v(x) \* u'(x) - u(x) \* v'(x)) / [v(x)]^2] = [(e^x - cos(x)) \* (1 + cos(x)) - (x + sin(x)) \* (e^x + sin(x))] / [(e^x - cos(x))^2]

y' = [(e^x - cos(x) + e^x\*cos(x) - x\*e^x - x\*sin(x) - sin(x)\*cos(x))] / [(e^x - cos(x))^2]

y' = [(2e^x\*cos(x) - x\*e^x - x\*sin(x) - sin(x)\*cos(x) - cos(x))] / [(e^x - cos(x))^2]

So, the derivative of the given function y = (x + sin(x))/(e^x - cos(x)) is:

y' = [(2e^x\*cos(x) - x\*e^x - x\*sin(x) - sin(x)\*cos(x) - cos(x))] / [(e^x - cos(x))^2]

(b)

To find the derivative of y with respect to x and then evaluate it at θ = π/2, we can use the chain rule and implicit differentiation. First, let's find dy/dθ using the given parametric equations for x and y.

Given:

x = a(θ + sinθ)

y = a(1 - cosθ)

Find dy/dθ.

Differentiate both x and y with respect to θ:

dx/dθ = a(1 + cosθ) (Derivative of θ + sinθ is 1 + cosθ)

dy/dθ = a(sinθ) (Derivative of 1 - cosθ is sinθ)

Find dy/dx using implicit differentiation.

We have x = a(θ + sinθ) and y = a(1 - cosθ). We can rewrite y in terms of x to use implicit differentiation:

y = a(1 - cosθ) → y = a(1 - √(1 - (x/a)^2))

Now, differentiate both sides with respect to x:

dy/dx = d/dx [a(1 - √(1 - (x/a)^2))]

To find dy/dx, we need to apply the chain rule.

Chain rule: d/dx [f(g(x))] = f'(g(x)) \* g'(x)

Let:

f(u) = a(1 - √u)

g(x) = 1 - (x/a)^2

Now, differentiate f(u) and g(x):

f'(u) = d/du [a(1 - √u)] = -a/(2√u)

g'(x) = d/dx [1 - (x/a)^2] = (-2x/a^2)

Now, apply the chain rule:

dy/dx = f'(g(x)) \* g'(x)

dy/dx = [-a/(2√(1 - (x/a)^2))] \* (-2x/a^2)

dy/dx = (a \* x) / (a^2√(1 - (x/a)^2))

Step 3: Evaluate dy/dx at θ = π/2.

Since we have x = a(θ + sinθ), when θ = π/2:

x = a(π/2 + sin(π/2))

x = a(π/2 + 1)

x = a(π/2 + 2/2)

x = a(π/2 + 1)

Now, substitute this value of x into dy/dx:

dy/dx = (a \* x) / (a^2√(1 - (x/a)^2))

dy/dx = (a \* [a(π/2 + 1)]) / (a^2√(1 - [a(π/2 + 1)/a]^2))

dy/dx = (a \* [a(π/2 + 1)]) / (a^2√(1 - [(π/2 + 1)]^2))

Now, simplify the expression:

dy/dx = a^2(π/2 + 1) / (a^2√(1 - [(π/2 + 1)]^2))

dy/dx = (π/2 + 1) / √(1 - [(π/2 + 1)]^2)

Therefore, [dy/dx]\_(θ=π/2) = (π/2 + 1) / √(1 - [(π/2 + 1)]^2) when θ = π/2.